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Application of the Caffeine in Antiviral Medications through Polynomials and Topological Descriptors

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Abstract: The two main sources of caffeine in the adult diet in North America are coffee (60 to 75%) and tea (15 to 30%), although caffeinated soft drinks and chocolate are primary sources of caffeine in kids' diets. An algorithm can generate topological indices, which are mathematical representations of molecules that can be applied to their structure. The degree-based topological indices for the aforementioned antiviral medications have been examined in the current work using a polynomial technique. Caffeine is a significant component in various medications, with different prescription (30 to 100 milligrams per tablet), over-the-counter (15 to 200 milligrams per tablet or pill), and capsule dosages. Topological indices are employed to describe the biological and physico-chemical properties of diverse chemical compounds. The adult acute fatal dosage of caffeine has been calculated to be 10 grams per person. While 6.5 grams of caffeine have been reported to be fatal, there are cases of patients who purportedly consumed 24 grams of caffeine and survived. We have computed various polynomials including Hosoya and Harary, distance-based topological indices, and other important degree-based topological indices.

Key Words: antiviral medicine; caffeine chemical structure; topological descriptors; Hosoya and Harary polynomials

I. INTRODUCTION

Pharmaceuticals and personal care products (PPCPs), food additives, and other consumer products have recently been identified as new pollutants resulting from human activity (Sophia and Lima, 2018). Due to potential risks to the environment and public health, pharmaceutical emerging contaminants (PECs) have become a significant concern, with caffeine being one of the most prevalent PECs found in various aquatic environments worldwide, including wastewater, rivers, groundwater, oceans, and even drinking water. It is a psycho-stimulant that is widely consumed [1].

Coffee is the primary dietary source of caffeine in adult diets in some European countries such as Sweden, Denmark, Finland, and Switzerland. Brewed coffee contains caffeine in the range of 56 to 100 milligrams per 100 milliliters. Tea and instant coffee follow with caffeine content ranging from 20 to 73 milligrams per 100 milliliters, and similar amounts are found in cola beverages (9-19 milligrams per 100 milliliters). Chocolate and cocoa products (e.g., chocolate candies) also contribute significantly to caffeine intake, with amounts around 3-5 milligrams per 100 grams. Moreover, caffeine is present in both prescription medications (30-100 milligrams per tablet) and non-prescription options (15 to 200 milligrams per tablet, pill, or capsule) [2]–[4].

Reported values for typical daily caffeine consumption from all sources in Canada range from approximately 2.4 mg/kg body weight for adults to 1.1 mg/kg for children aged 5 to 18 [5]. In a study involving adults (481 males and females, ages 30 to 75) in southern Ontario, Canada, caffeine consumption ranged from 288 to 426 mg per day, equivalent to 4.5 to 6.5 mg/kg body weight in a 65-kilogram individual.

Average daily caffeine intake in the general population is around 3 mg/kg body weight in the USA, 4 mg/kg in the UK, and 7 mg/kg in Denmark. Advanced consumers may consume doses ranging from 5 to 15 mg/kg body weight. The recommended daily caffeine intake for children varies, with values of 1 mg/kg body weight in the USA, 3 mg/kg in the UK, and 2.5 mg/kg in Denmark [5].

Caffeine's elimination half-life ranges from 3 to 7 hours and is influenced by factors such as sex, age, oral contraceptive use, pregnancy, and smoking. Women tend to have shorter half-lives than men (20-30% shorter), and newborns have a half-life of 50-100 hours, gradually approaching that of adults by six months of age. The half-life is higher in females using oral contraceptives compared to ovulatory females. Pregnancy also affects caffeine metabolism, with

the half-life increasing from 4 hours in the first trimester to 18 hours in the third trimester. Cigarette smoking accelerates caffeine excretion by nearly threefold [2], [6].

Caffeine is rapidly absorbed into the bloodstream through the gastrointestinal tract. Blood levels peak within 1 to 1.5 hours after consumption. It crosses the blood-brain barrier, placenta, and breast milk, and has been detected in semen. Caffeine metabolism primarily occurs in the liver, resulting in the formation of metabolites such as 1-methylxanthine and 1-methyluric acid. Only 1 to 5% of consumed caffeine is excreted unchanged in urine. Caffeine metabolism in newborns is impaired, with around 85% excreted. While cases of death from excessive caffeine use are rare, an adult's acute fatal dose has been estimated at 10 grams per person. There have been reports of survival even after consuming as much as 24 grams of caffeine [7].

Consider a simple graph $G = (V, E)$, where the Hosoya polynomial is defined as $H(\xi, \lambda) = \sum_{(u_0,u_1)\subseteq V(\xi)} \lambda^{d(u_0,u_1)}$, and $d(u_0, u_1)$ denotes the distance between vertices u_0 and u_1. Cactus graphs are characterized by not having edges that span more than one cycle, making them connected. In a study [8], the Hosoya polynomial of several cactus chains was computed, leading to the determination of WI and hyper-WI for these chains.

Topological indices are molecular descriptors that capture various aspects of a molecule's structure using graph theory. The WI, an early topological index, was introduced by Wiener in 1947 as the "path number." The Hosoya polynomial is central to finding the WI. In another study [9], the Hosoya and Harary polynomials were computed for different network structures, including triangular oxide and silicate networks, along with various distance-based topological indices.

The Hosoya polynomial, introduced by Hosoya in 1988, has the remarkable ability to recover nearly all distance-based topological indices. In a study [10], the authors provide the general closed form of the Hosoya polynomial for a linearly concatenated benzene molecule. They show that the Hosoya polynomial can be used to predict various indices for compounds containing B_n, providing insights into pharmacological, chemical, and physical characteristics.

The paper also introduces topological indices, particularly the Hosoya polynomial, as a mathematical tool to characterize molecular structures in chemical compounds.

II. BASIC NOTIONS AND DEFINITIONS

The Hosoya polynomial of a connected graph ξ is defined as,

$$H(\xi, \lambda) = \sum_{(u_0,u_1)\subseteq V(\xi)} \lambda^{d(u_0,u_1)},$$

where the vertices u and v's distance is indicated by the symbol $d(u, v)$ [11].

The Harary polynomial of a connected graph ξ is defined as [12],

$$h(\xi, \lambda) = \sum_{(u_0,u_1)\subseteq V(\xi)} \frac{1}{d(u_0, u_1)} \lambda^{(u_0,u_1)}.$$

The sum of the distances between all pairs of vertices in a connected graph ξ is the Wiener index. The mathematical formula for the Wiener index is [13],

$$W(\xi) = \sum_{(u_0,u_1)\subseteq V(\xi)} d(u_0, u_1).$$

The hyper-Wiener index $WW(\xi)$ is defined as,

$$WW(\xi) = \frac{1}{2} \sum_{(u_0,u_1)\subseteq V(\xi)} d(u_0, u_1) + \frac{1}{2} \sum_{(u_0,u_1)\subseteq V(\xi)} d^2(u_0, u_1).$$

Note that the first derivative of the Hosoya polynomial at $\lambda = 1$ is equal to the Wiener index [14],

$$W(\xi) = (H(\xi, \lambda))'|_{\lambda=1}.$$

Also, we have the following relation,

$$WW(\xi) = \frac{1}{2}(\lambda H(\xi, \lambda))'\prime|_{\lambda=1}.$$

In 1972, Gutman and Trinajstic [15] introduced the first Zagreb and second Zagreb indices as,

$$M_1(\wp) = \sum_{mn\in E(\wp)} (d_m + d_n),$$

and

$$M_2(\wp) = \sum_{mn\in E(\wp)} (d_m \times d_n).$$

The hyper-Zagreb index is defined as,

$$HM(\wp) = \sum_{mn\in E(\wp)} (d_m + d_n)^2.$$

The redefined 3^{rd} Zagreb index is defined as,

$$ReZG_3(\xi) = \sum_{mn\in E(\xi)} (d_m d_n)(d_m + d_n).$$

The forgotten topological index is defined as,

$$F(\xi) = \sum_{gh\in E(\xi)} (d_g^2 + d_h^2).$$

The first and second Gourava indices are defined as,

$$GO_1(\xi) = \sum_{gh\in E(\xi)} [(d_g + d_h) + (d_g \times d_h)],$$

and

$$GO_2(\xi) = \sum_{gh\in E(\xi)} [(d_g + d_h)(d_g \times d_h)].$$

The first and second hyper Gourava indices are defined as,

$$HGO_1(\xi) = \sum_{gh\in E(\xi)} [(d_g + d_h) + (d_g \times d_h)]^2,$$

and

$$HGO_2(\xi) = \sum_{gh\in E(\xi)} [(d_g + d_h)(d_g \times d_h)]^2.$$

The general redefined Zagreb index is defined as,

$$ReZ_{(\delta_1,\delta_2)}(\xi) = \sum_{m,n\in E(\xi)} [d_m \times d_n]^{\delta_1} [d_m + d_n]^{\delta_2},$$

where δ_1 and δ_2 are real numbers. Opting δ_1=0 and $\delta_2 = \delta_1$, $\delta_2 \neq 0$ give rise to the general sum-connectivity index that is defined as,

$$\chi_{\delta_1}(\xi) = \sum_{mn \in E(\xi)} [d_m + d_n]^{\delta_1}.$$

Whereas for $\delta_1 \neq 0$ and $\delta_2 = 0$, the general Randic-connectivity index R_{δ_1} is obtained which is defined as,

$$R_{\delta_1}(\xi) = \sum_{mn \in E(G)} [d_m \times d_n]^{\delta_1}.$$

The first Zagreb polynomial and second Zagreb polynomial are defined as,

$$M_1(\xi, \lambda) = \sum_{vw \in E(\xi)} \lambda^{d_v + d_w},$$

and

$$M_2(\xi, \lambda) = \sum_{vw \in E(\xi)} \lambda^{d_v d_w}.$$

The Hyper-Zagreb polynomial is defined as,

$$HM(\xi, \lambda) = \sum_{vw \in E(\xi)} \lambda^{[d_v + d_w]^2}.$$

General redefined Zagreb Polynomial is defined as,

$$ReZG_{(\delta_1, \delta_2)}(\xi, \lambda) = \sum_{vw \in E(\xi)} \lambda^{[d_v \times d_w]^{\delta_1} [d_v + d_w]^{\delta_2}}.$$

The redefined 3^{rd} Zagreb polynomial is defined as,

$$ReZG_3(\xi, \lambda) = \sum_{vw \in E(\xi)} \lambda^{(d_v \times d_w)(d_v + d_w)}.$$

The general sum-connectivity polynomial is defined as,

$$\chi_{\delta_1}(\xi, \lambda) = \sum_{vw \in E(\xi)} \lambda^{[d_v + d_w]^{\delta_1}}.$$

The general Randic-connectivity polynomial is defined as,

$$R_{\delta_1}(\xi, \lambda) = \sum_{vw \in \xi(G)} \lambda^{[d_v \times d_w]^{\delta_1}}.$$

III. RESULTS AND DISCUSSIONS

In this section, we presented some important results related to Caffeine chemical structure like Hosoya and Harary polynomials and some useful topological indices. The graphical representation of chemical structure of Caffeine is presented in Figure 1 and the vertex partition of Caffeine chemical structure is given in Table 1. Moreover, the edge partition of Caffeine chemical structure is given in Table 2. From the Figure 1 and Tables 1 and 2, we have following results for Caffeine chemical structure;

TABLE 1: The vertex partition of Caffeine chemical structure

d_g where $g \in V(\xi)$	vertices
1	$10n + 2$
2	$2n - 1$
3	$8n$
4	$3n$

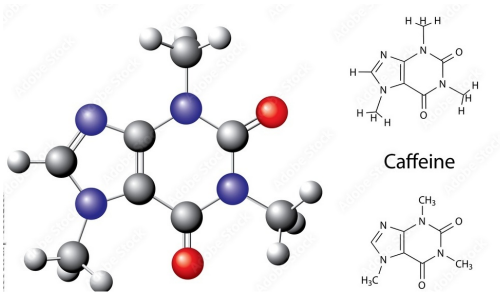

FIGURE 1: Chemical structure of Caffeine

TABLE 2: The edge partition of Caffeine chemical structure

(d_g, d_h) where $gh \in E(\xi)$	No. of edges
$E_1(1,3)$	$2n + 1$
$E_2(1,4)$	$8n + 1$
$E_3(2,3)$	$3n - 1$
$E_4(2,4)$	$n - 1$
$E_5(3,3)$	$8n$
$E_6(3,4)$	$3n$

The Hosoya polynomial is,

$$H(\xi, \lambda) = 25\lambda + 43\lambda^2 + 46\lambda^3 + 57\lambda^4 + 42\lambda^5 + 39\lambda^6 + 24\lambda^7.$$

The Harary polynomial is,

$$h(\xi) = 25\lambda + \frac{1}{2}43\lambda^2 + \frac{1}{3}46\lambda^3 + \frac{1}{4}57\lambda^4 + \frac{1}{5}42\lambda^5 + \frac{1}{6}39\lambda^6 + \frac{1}{7}24\lambda^7.$$

The Wiener index is,

$$W(\xi) = 1089.$$

The Hyper-Wiener index is,

$$WW(\xi) = 3121.$$

Moreover, from Tables 1 and 2, the following results for the Caffeine chemical structure can be obtained immediately,

1) The first Zagreb index: $M_1(\wp) = 138n - 2$.
2) The second Zagreb index: $M_2(\wp) = 172n - 7$.
3) The Hyper-Zagreb index: $HM(\wp) = 778n - 20$.
4) The forgotten topological index: $F(\xi) = 434n - 6$.
5) The first gourava index: $GO_1(\xi) = 340n - 39$.
6) The second gourava index: $GO_2(\xi) = 1006n - 46$.
7) The first hyper-gourava index: $HGO_1(\xi) = 4188n - 187$.
8) The second hyper-gourava index: $HGO_2(\xi) = 52988n - 2660$.
9) Redefined general Zagreb index: $ReZG_{(\delta_1,\delta_2)} = (2n+1)[3]^{\delta_1}[4]^{\delta_2} + (8n+1)[4]^{\delta_1}[5]^{\delta_2} + (3n-1)[6]^{\delta_1}[5]^{\delta_2} + (n-1)[8]^{\delta_1}[6]^{\delta_2} + (8n)[9]^{\delta_1}[6]^{\delta_2} + (3n)[12]^{\delta_1}[7]^{\delta_2}$.
10) Redefined 3^{rd} Zagreb index: $ReZG_3 = 1006n - 46$.
11) General sum-connectivity index: $\chi_{\delta_1}(\xi) = (2n+1)[4]^{\delta_1} + (8n+1)[5]^{\delta_1} + (11n)[5]^{\delta_1} + (9n-1)[6]^{\delta_1} + 3n[7]^{\delta_1}$.

12) General Randic-connectivity index: $R_{\delta_1}(\xi) = (2n+1)[3]^{\delta_1} + (8n+1)[4]^{\delta_1} + (3n-1)[6]^{\delta_1} + (n-1)[8]^{\delta_1} + (8n)[9]^{\delta_1} + (3n)[12]^{\delta_1}$.

Similarly, the following polynomials for the Caffeine chemical structure can be calculated,

1) First Zagreb polynomial: $M_1(\xi,\lambda) = n(2\lambda^4 + 11\lambda^5 + 9\lambda^6 + 3\lambda^7) + \lambda^4 - \lambda^6$.
2) Second Zagreb polynomial: $M_2(\xi,\lambda) = n(2\lambda^3 + 8\lambda^4 + 3\lambda^6 + \lambda^8 + 8\lambda^9 + 3\lambda^{12}) + \lambda^3 + \lambda^4 - \lambda^6 - \lambda^8$.
3) Hyper-Zagreb polynomial: $HM(\xi,\lambda) = n(2\lambda^{16} + 11\lambda^{25} + 9\lambda^{36} + 3\lambda^{49}) + \lambda^{16} - \lambda^{36}$.
4) Redefined 3^{rd} Zagreb polynomial: $ReZG_3(\xi,\lambda) = n(2\lambda^{12} + 8\lambda^{20} + 3\lambda^{30} + \lambda^{48} + 8\lambda^{54} + 3\lambda^{84}) + \lambda^{12} + \lambda^{20} - \lambda^{30} - \lambda^{48}$.
5) General sum-connectivity polynomial: $\chi_{\delta_1}(\xi,\lambda) = n(2\lambda^{[4]^{\delta_1}} + 11z^{[5]^{\delta_1}} + 9\lambda^{[6]^{\delta_1}} + 3\lambda^{[7]^{\delta_1}}) + \lambda^{[4]^{\delta_1}} - \lambda^{[6]^{\delta_1}}$.
6) General Randic-connectivity polynomial: $R_{\delta_1}(\xi,\lambda) = n(2\lambda^{[3]^{\delta_1}} + 8\lambda^{[4]^{\delta_1}} + 3\lambda^{[6]^{\delta_1}} + \lambda^{[8]^{\delta_1}} + 8\lambda^{[9]^{\delta_1}} + 3\lambda^{[12]^{\delta_1}}) + \lambda^{[3]^{\delta_1}} + \lambda^{[4]^{\delta_1}} - \lambda^{[6]^{\delta_1}} - \lambda^{[8]^{\delta_1}}$.

IV. CONCLUSION

In this study, the topological features of the caffeine chemical structure, which is present in various sources such as kids' products, chocolate, and medicines, are explored. The ability of caffeine to cross the blood-brain barrier, traverse the placenta, enter the foetus' amniotic fluid and breast milk, and undergo metabolism primarily in the liver is investigated in the research paper. Various distance-based and degree-based topological indices are computed for caffeine's chemical structure. Hosoya polynomial, Harary polynomial, and degree-based polynomials are assessed using graphical representations. The resulting polynomial expressions are used to calculate topological indices for each structure. This research also examines degree-based topological indices for antiviral medications, employing polynomial techniques. The computed topological indices contribute to understanding diverse biological and physicochemical features of chemical compounds. The findings support positive assumptions in chemical and pharmaceutical engineering presentations.

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